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Mach Disk Location in Jets in Co-Flowing Airstreams

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Nomenclature

M_j = Mach number at nozzle exit
M_∞ = freestream Mach number
P_j = static pressure at nozzle exit
P_∞ = freestream static pressure
r_b = base radius of body
r_j = nozzle exit radius
x_s = axial location of Mach disk measured downstream from nozzle exit plane
γ = ratio of specific heats
θ_n = nozzle exit angle

Introduction

SYSTEM design requirements frequently necessitate a prediction of the structure of the plume formed by the exhaust of an underexpanded nozzle flow into the surrounding environment. The difficulty of this prediction is a function of the degree of underexpansion involved, the downstream distance over which the plume is to be defined, and the motion of the environment relative to the plume. A condition of particular interest in flight situations where exhaust plume signal interference and/or infrared signature must be assessed, involves the exhaust of a moderately under-expanded flow into a co-flowing airstream. In such cases the structure of a plume must be defined in regions dominated by gas dynamic effects in the vicinity of the nozzle exit plane, and by the effects of turbulent mixing, and frequently chemistry, further downstream. Although no practical means are presently available for the general computation of such complex flowfields, efforts continue in the development of flow models which attempt to account for the influence of those effects considered to have a major influence on the plume structure.[1–4] This Note concerns one particular aspect of the prediction problem, namely, the axial location of the Mach disk in an underexpanded jet in a co-flowing airstream.

Disk Location for Exhaust into a Still Environment

The foundation of the present work lies in a review of past research concerning the Mach disk location in plumes formed following the exhaust of an underexpanded flow into a still environment. Reporting on an extensive series of experimental and theoretical studies, Love et al.[5] found that the axial location of the Mach disk is not strongly affected by variations in nozzle exit angle. For a given ratio of specific heats, then, the axial Mach disk location (nondimensionalized in terms of the nozzle exit radius) could be determined by specification of the exit-to-ambient pressure ratio and the exit Mach number.

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A later study by Adamson and Nicholls[6] confirmed the findings of Love et al., but more importantly, resulted in a method for the analytical prediction of the Mach disk location which was found to be in substantial agreement with the experiment. In summary, they hypothesized that the Mach disk would be found at a point along the nozzle axis where the compression of the expanded plume flow through a normal shock wave resulted in the elevation of the local static pressure to the ambient value. To implement this hypothesis, they noted that the plume flowfield in the region bounded by the nozzle exit plane, the intercepting shock wave, and the Mach disk, is for a given ratio of specific heats, solely a function of the flow conditions at the nozzle exit plane.[7] Thus, they were able to utilize the results of an earlier study for the flowfield produced by exhaust from a near sonic orifice into a vacuum[7] to predict the axial location of the Mach disk as a function of exit-to-ambient pressure ratio and exit Mach number for a ratio of specific heats of 1.4.

The experimental work of Lewis and Carlson[8] included a consideration of the effect of the ratio of specific heats on the Mach disk location, and, in general, greatly simplified the prediction procedure. They found that the axial locations of the Mach disk observed in their studies, as well as those that had been determined by earlier investigators, could be correlated by using the equation, $x_s/r_j = 1.38 M_j(\gamma p_j/p_\infty)^{0.5}$.

Disk Location for Exhaust into a Co-Flowing Airstream

Consider, now, the situation in which the underexpanded flow exhausts into a co-flowing airstream. Schlieren photographs presented by Love et al.[5] for the case in which the external flow is supersonic reveal the absence of shock waves in the external flow downstream of the recompression shock located at the end of the first plume wavelength. This suggests, then, that the pressure in the plume, and in the external flow, is nearly ambient downstream of the first plume wavelength. Further, measurements by Furby[9] demonstrate that a similar condition prevails for exhaust into a still environment. Thus, it is reasonable

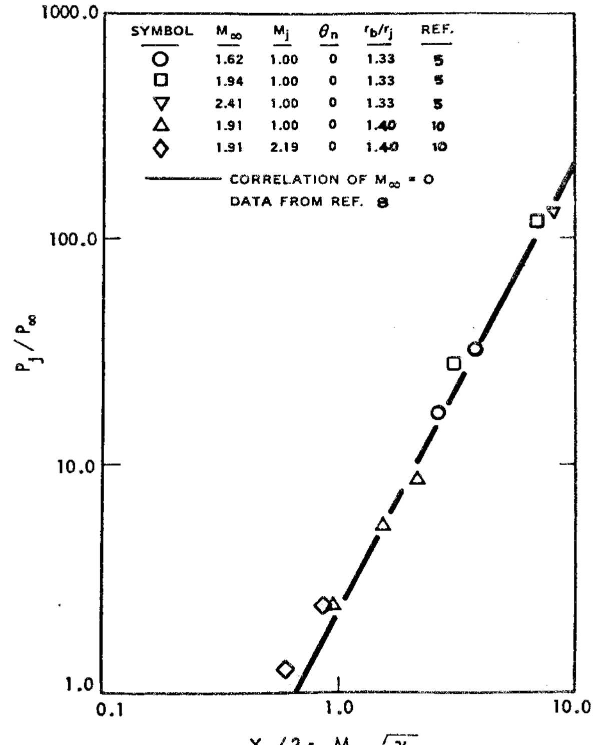

Fig. 1 Correlation of Mach disk location in plumes and in the presence of co-flowing streams.

to assume that the method of Adamson and Nicholls for the prediction of the Mach disk location in plumes formed in still air would apply with similar accuracy for exhaust into a co-flowing airstream. Finally, since the axial Mach number distribution upstream of the Mach disk is independent of the external condition, we conclude from these arguments that the axial location of the Mach disk in a plume contained in a co-flowing airstream would, for the same exit-to-ambient pressure ratio, be identical to its location in a plume formed in still air (the limitations of this conclusion will be discussed later).

To test the validity of the conclusion reached, Mach disk locations were extracted from Schlieren photographs of plumes formed in the presence of supersonic external flows.[5,10] These were then compared with the results of the experiments for exhaust into still air as represented by Lewis and Carlson's empirical correlation. The result, presented in Fig. 1, supports the conclusion that, over the range of conditions investigated, the axial position of the Mach disk is essentially independent of the Mach number in the external flow.

Limitations on Conclusion

1) From the results presented, the height of the nozzle base appears to have no influence on the Mach disk location. Such would not be true in a situation where the first plume wavelength were totally immersed in the base flow. Under this condition, the Mach disk would be found at a location corresponding to a pressure ratio equal to the exit-to-base pressure ratio. The latter represents, however, an infrequently encountered flow situation. 2) It cannot be assumed that a Mach disk, if found in a plume in still air, will always be found in the presence of external flow. While the external flow has no influence on the axial Mach number distribution in the region previously described, it does have an influence on the trajectory of the intercepting shock wave. Thus, at a fixed pressure ratio, the intercepting shock is displaced toward the axis of symmetry as the freestream Mach number is increased. If the freestream Mach number is sufficiently high, the intercepting shock can reach the axis upstream of the point predicted for the Mach disk location. Under this condition, a Mach disk will not form, and a regular reflection will occur.

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Influence of Subsonic Potential Flow on the Buckling of Thin Panels under Edge Compression

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THIS Note was prompted by recent publications[1,2] dealing with the title problem on the basis of ad-hoc approximate aerodynamic theories of a rather intuitive character. Here, the aerodynamic operator is derived from the linearized theory of subsonic potential flow and the solution is obtained by Galerkin's method.

Consider a thin elastic panel of infinite width simply supported along its edges $\bar{x} = 0$ and $\bar{x} = l$ and loaded by uniformly distributed compressive forces N_x. The panel is set in an infinite plane $z = 0$ and placed in a fluid flow of subsonic velocity U in the $\bar{x}$ direction. To investigate the static stability of the panel (static divergence precedes the onset of flutter in subsonic flow, e.g.,[3]) we impose on it a small lateral deflection $w(x)$ thereby changing the dynamic pressure of the ambient flow by $\Delta p(x)$ and seeking the values of N_x and U at which the deflected panel is in equilibrium.

The equation of equilibrium is

$$D\frac{d^4w}{dx^4} + N_x l^2 \frac{d^2w}{dx^2} + \Delta p(x) l^4 = 0 \tag{1}$$

subject to the boundary conditions:

$$w = 0,\ w'' = 0 \quad \text{at} \quad x = 0, 1 \tag{2}$$

where x is the dimensionless streamwise coordinate ($x = \bar{x}/l$) and D the bending stiffness of the panel.

The dynamic pressure on the upper surface of the panel is given by the Bernoulli equation

$$p_u = -\frac{\rho U}{l}\frac{\partial \phi_u}{\partial x}\bigg|_{z=0^+} \tag{3}$$

where ρ is the mass density of the fluid and ϕ_u the perturbation velocity potential in the upper half space ($z \geqq 0$). For a subsonic ($M < 1$), compressible inviscid irrotational fluid, $\phi_u(x, z)$ is determined by solving the equation

$$(1 - M^2)\frac{\partial^2 \phi_u}{\partial x^2} + \frac{\partial^2 \phi_u}{\partial z^2} = 0 \tag{4}$$

subject to the boundary conditions:

$$\frac{\partial \phi_u}{\partial z}\bigg|_{z=0^+} = \begin{cases} U\dfrac{dw}{dx} & \text{for} \quad 0 \leqq x \leqq 1 \\ 0 & \text{elsewhere} \end{cases} \tag{5}$$

and

$$\partial\phi_u/\partial z,\ \partial\phi_u/\partial x \to 0 \quad \text{at} \quad z \to \infty \tag{6}$$

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